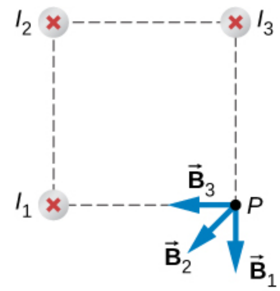

The magnetic field in the *x*-direction has contributions from wire 3 and the *x*-component of wire 2:

$$B_{\text{net } x} = -4 \times 10^{-5}\,\text{T} - 2.83 \times 10^{-5}\,\text{T}\cos(45°) = -6 \times 10^{-5}\,\text{T}.$$

The *y*-component is similarly the contributions from wire 1 and the *y*-component of wire 2:

$$B_{\text{net } y} = -4 \times 10^{-5}\,\text{T} - 2.83 \times 10^{-5}\,\text{T}\sin(45°) = -6 \times 10^{-5}\,\text{T}.$$

Therefore, the net magnetic field is the resultant of these two components:

$$B_{\text{net}} = \sqrt{B^2_{\text{net } x} + B^2_{\text{net } y}}$$
$$B_{\text{net}} = \sqrt{(-6 \times 10^{-5}\,\text{T})^2 + (-6 \times 10^{-5}\,\text{T})^2}$$
$$B_{\text{net}} = 8 \times 10^{-5}\,\text{T}.$$

Significance

The geometry in this problem results in the magnetic field contributions in the *x*- and *y*-directions having the same magnitude. This is not necessarily the case if the currents were different values or if the wires were located in different positions. Regardless of the numerical results, working on the components of the vectors will yield the resulting magnetic field at the point in need.

12.3 Check Your Understanding Using **Example 12.3**, keeping the currents the same in wires 1 and 3, what should the current be in wire 2 to counteract the magnetic fields from wires 1 and 3 so that there is no net magnetic field at point P?

12.3 | Magnetic Force between Two Parallel Currents

Learning Objectives

By the end of this section, you will be able to:

- Explain how parallel wires carrying currents can attract or repel each other
- Define the ampere and describe how it is related to current-carrying wires
- Calculate the force of attraction or repulsion between two current-carrying wires

You might expect that two current-carrying wires generate significant forces between them, since ordinary currents produce magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to define the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long, straight, and parallel conductors separated by a distance *r* can be found by applying what we have developed in the preceding sections. **Figure 12.9** shows the wires, their currents, the field created by one wire, and the consequent force the other wire experiences from the created field. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance *r* is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \tag{12.9}$$

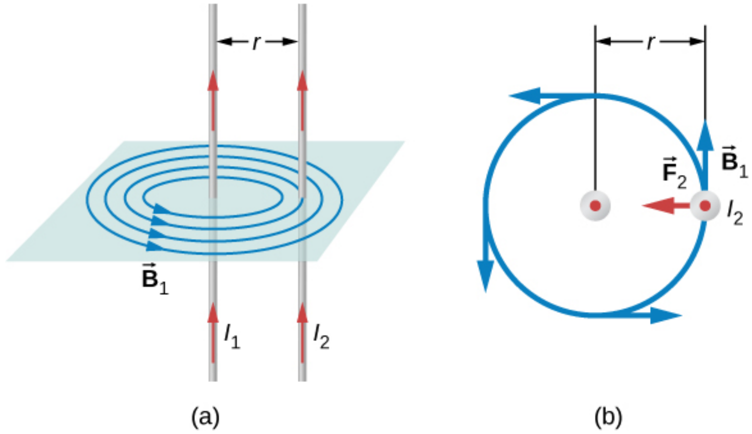

Figure 12.9 (a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by right-hand rule (RHR)-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for wire 1. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

This field is uniform from the wire 1 and perpendicular to it, so the force F_2 it exerts on a length *l* of wire 2 is given by $F = IlB\sin\theta$ with $\sin\theta = 1$:

$$F_2 = I_2 lB_1. \tag{12.10}$$

The forces on the wires are equal in magnitude, so we just write *F* for the magnitude of F_2. (Note that $\vec{\mathbf{F}}_1 = -\vec{\mathbf{F}}_2$.) Since the wires are very long, it is convenient to think in terms of *F/l*, the force per unit length. Substituting the expression for B_1 into **Equation 12.10** and rearranging terms gives

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}. \tag{12.11}$$

The ratio *F/l* is the force per unit length between two parallel currents I_1 and I_2 separated by a distance *r*. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and other plasmas. The force exists whether the currents are in wires or not. It is only apparent if the overall charge density is zero; otherwise, the Coulomb repulsion overwhelms the magnetic attraction. In an electric arc, where charges are moving parallel to one another, an attractive force squeezes currents into a smaller tube. In large circuit breakers, such as those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The definition of the ampere is based on the force between current-carrying wires. Note that for long, parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

$$\frac{F}{l} = \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)(1\text{ A})^2}{(2\pi)(1\text{ m})} = 2 \times 10^{-7}\text{ N/m}. \tag{12.12}$$

Since μ_0 is exactly $4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}$ by definition, and because $1\text{ T} = 1\text{ N/(A}\cdot\text{m)},$ the force per meter is exactly

2×10^{-7} N/m. This is the basis of the definition of the ampere.

Infinite-length wires are impractical, so in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1\text{ C} = 1\text{ A} \cdot \text{s}$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

Example 12.4

Calculating Forces on Wires

Two wires, both carrying current out of the page, have a current of magnitude 5.0 mA. The first wire is located at (0.0 cm, 3.0 cm) while the other wire is located at (4.0 cm, 0.0 cm) as shown in **Figure 12.10**. What is the magnetic force per unit length of the first wire on the second and the second wire on the first?

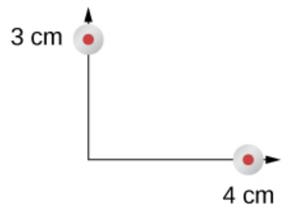

Figure 12.10 Two current-carrying wires at given locations with currents out of the page.

Strategy

Each wire produces a magnetic field felt by the other wire. The distance along the hypotenuse of the triangle between the wires is the radial distance used in the calculation to determine the force per unit length. Since both wires have currents flowing in the same direction, the direction of the force is toward each other.

Solution

The distance between the wires results from finding the hypotenuse of a triangle:

$$r = \sqrt{(3.0\text{ cm})^2 + (4.0\text{ cm})^2} = 5.0\text{ cm}.$$

The force per unit length can then be calculated using the known currents in the wires:

$$\frac{F}{l} = \frac{\left(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A}\right)\left(5 \times 10^{-3}\,\text{A}\right)^2}{(2\pi)(5 \times 10^{-2}\,\text{m})} = 1 \times 10^{-10}\text{ N/m}.$$

The force from the first wire pulls the second wire. The angle between the radius and the *x*-axis is

$$\theta = \tan^{-1}\left(\frac{3\text{ cm}}{4\text{ cm}}\right) = 36.9°.$$

The unit vector for this is calculated by

$$\cos(36.9°)\,\hat{\mathbf{i}} - \sin(36.9°)\,\hat{\mathbf{j}} = 0.8\,\hat{\mathbf{i}} - 0.6\,\hat{\mathbf{j}}.$$

Therefore, the force per unit length from wire one on wire 2 is

$$\frac{\vec{\mathbf{F}}}{l} = (1 \times 10^{-10}\text{ N/m}) \times (0.8\,\hat{\mathbf{i}} - 0.6\,\hat{\mathbf{j}}) = (8 \times 10^{-11}\,\hat{\mathbf{i}} - 6 \times 10^{-11}\,\hat{\mathbf{j}})\text{ N/m}.$$

The force per unit length from wire 2 on wire 1 is the negative of the previous answer:

$$\frac{\vec{\mathbf{F}}}{l} = (-8 \times 10^{-11}\,\hat{\mathbf{i}} + 6 \times 10^{-11}\,\hat{\mathbf{j}})\text{N/m}.$$

Significance

These wires produced magnetic fields of equal magnitude but opposite directions at each other's locations. Whether the fields are identical or not, the forces that the wires exert on each other are always equal in magnitude and opposite in direction (Newton's third law).

12.4 Check Your Understanding Two wires, both carrying current out of the page, have a current of magnitude 2.0 mA and 3.0 mA, respectively. The first wire is located at (0.0 cm, 5.0 cm) while the other wire is located at (12.0 cm, 0.0 cm). What is the magnitude of the magnetic force per unit length of the first wire on the second and the second wire on the first?

12.4 | Magnetic Field of a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Explain how the Biot-Savart law is used to determine the magnetic field due to a current in a loop of wire at a point along a line perpendicular to thep lane of the loop.
- Determine the magnetic field of an arc of current.

The circular loop of **Figure 12.11** has a radius *R*, carries a current *I*, and lies in the *xz*-plane. What is the magnetic field due to the current at an arbitrary point *P* along the axis of the loop?

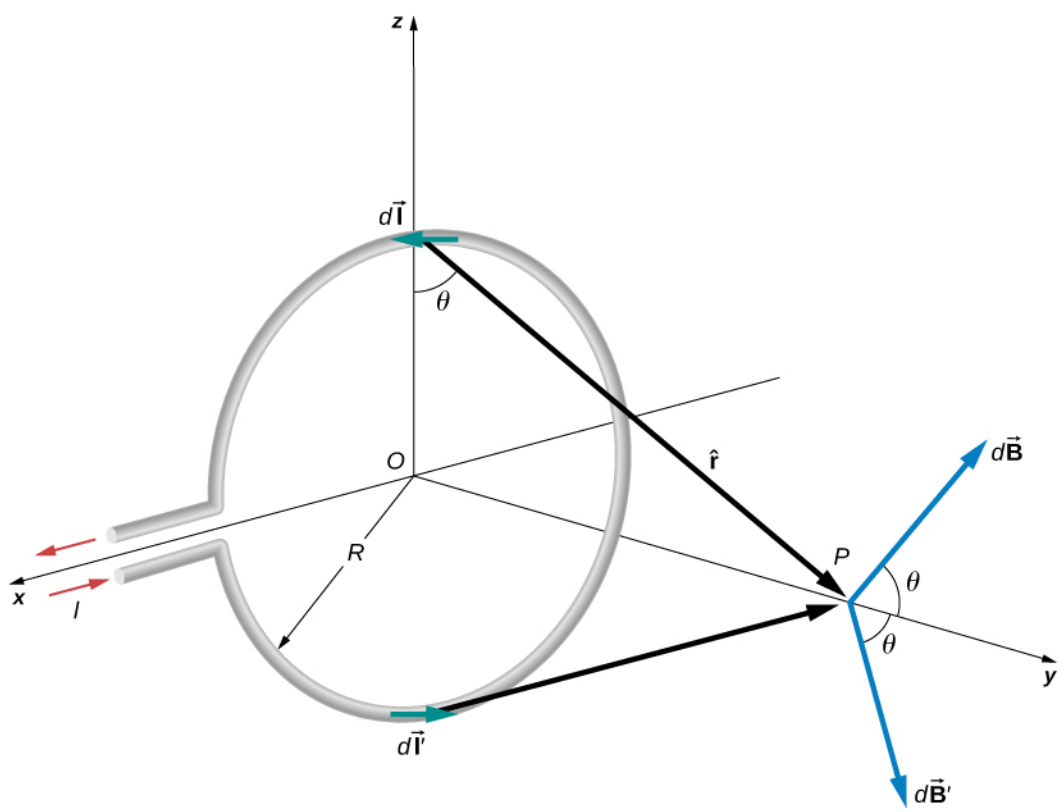

Figure 12.11 Determining the magnetic field at point *P* along the axis of a current-carrying loop of wire.

We can use the Biot-Savart law to find the magnetic field due to a current. We first consider arbitrary segments on opposite sides of the loop to qualitatively show by the vector results that the net magnetic field direction is along the central axis